Sujet A

TS 1 IRIS : Devoir N° 4

L'usage de la calculette et du formulaire est interdit dans ce devoir.

Les exercices peuvent être traités dans un ordre quelconque,
mais il faut indiquer leur numéro.

Calculer les intégrales suivantes

1) $$I = \int_0^1 \left(t + \frac{1}{3} + e^t\right) dt$$

2) $$I = \int_0^{\frac{\pi}{6}} \cos(3t)\, dt$$

3) $$I = \int_0^1 \frac{5}{t+1}\, dt$$

4) $$I = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} \frac{\cos(t)}{\sin(t)}\, dt$$ On pose le changement de variable : $u = \sin(t)$

5) $$I = \int_0^1 \frac{t+6}{t^2+9}\, dt$$

6) $$I = \int_{-1}^1 (2\,t+3)\, e^t\, dt$$

7) $$I = \int_2^3 \frac{t+2}{t^2-1}\, dt$$

8) $$I = \int_0^1 \frac{8\,t+14}{(t+1)(t-2)(t+3)}\, dt$$

9) $$I = \int_0^{\pi} \sin(3t)\, e^{-t}\, dt$$

10) $$I = \int_1^e \frac{1}{t\,\big(2+\ln(t)\big)}\, dt$$ On pose le changement de variable : $u = 2+\ln(t)$

Sujet B

TS 1 IRIS : Devoir N° 4

L'usage de la calculette et du formulaire est interdit dans ce devoir.

Les exercices peuvent être traités dans un ordre quelconque,
mais il faut indiquer leur numéro.

Calculer les intégrales suivantes

1) $$I = \int_0^1 \left(t + e^t + \frac{1}{2}\right) dt$$

2) $$I = \int_0^{\frac{\pi}{6}} \sin(3t)\, dt$$

3) $$I = \int_0^{\frac{\pi}{6}} \frac{\sin(t)}{\cos(t)}\, dt$$ On pose le changement de variable : $u = \cos(t)$

4) $$I = \int_0^1 \frac{7}{t+2}\, dt$$

5) $$I = \int_0^1 \frac{t+4}{t^2+4}\, dt$$

6) $$I = \int_{-1}^1 (3\,t+1)\, e^t\, dt$$

7) $$I = \int_2^3 \frac{t-3}{t^2-1}\, dt$$

8) $$I = \int_0^{\pi} \cos(2t)\, e^{-t}\, dt$$

9) $$I = \int_{-1}^1 \frac{6\,t-8}{(t+2)(t-2)(t-3)}\, dt$$

10) $$I = \int_1^e \frac{1}{t\,\big(\ln(t)+3\big)}\, dt$$ On pose le changement de variable : $u = \ln(t)+3$

TS 1 IRIS : Devoir N° 4 Sujet A (Solution)

Calculer les intégrales suivantes

1) $$I = \int_0^1 \left(t + \frac{1}{3} + e^t\right) dt = \left[\frac{t^2}{2} + \frac{t}{3} + e^t\right]_0^1 = \left(\frac{1}{2} + \frac{1}{3} + e\right) - (1) \qquad \boxed{I = e - \frac{1}{6}}$$

2) $$I = \int_0^{\frac{\pi}{6}} \cos(3t)\, dt = \left[\frac{\sin(3t)}{3}\right]_0^{\frac{\pi}{6}} = \left(\frac{1}{3}\right) - (0) \qquad \boxed{I = \frac{1}{3}}$$

3) $$I = \int_0^1 \frac{5}{t+1}\, dt = \Big[5\ln(t+1)\Big]_0^1 = 5\ln(2) - 5\ln(1) \qquad \boxed{I = 5\ln(2)}$$

4) $$I = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} \frac{\cos(t)}{\sin(t)}\, dt \qquad \text{On pose :} \quad u = \sin(t) \quad \text{donc :} \quad du = \cos(t)dt$$

$$I = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} \frac{\cos(t)}{\sin(t)}\, dt = \int_1^{\frac{\sqrt{3}}{2}} \frac{1}{u}\, du$$
$$= \Big[\ln(u)\Big]_1^{\frac{\sqrt{3}}{2}} = \left(\ln\left(\frac{\sqrt{3}}{2}\right)\right) - \Big(\ln(1)\Big) \qquad \boxed{I = \frac{1}{2}\,\ln\left(\frac{3}{4}\right)}$$

5) $$I = \int_0^1 \frac{t+6}{t^2+9}\, dt = \int_0^1 \left(\frac{1}{2}\frac{2t}{t^2+9} + \frac{6}{t^2+9}\right) dt = \left[\frac{1}{2}\ln(t^2+9) + \frac{6}{3}\arctan\left(\frac{t}{3}\right)\right]_0^1$$

$$I = \left(\frac{1}{2}\ln(10) + 2\arctan\left(\frac{1}{3}\right)\right) - \left(\frac{1}{2}\ln(9) + 2\arctan(0)\right)$$

$$\boxed{I = \frac{1}{2}\ln\left(\frac{10}{9}\right) + 2\arctan\left(\frac{1}{3}\right)}$$

6) $$I = \int_{-1}^1 (2\,t + 3)\, e^t\, dt$$

Intégration par partie : $\left| \begin{array}{l} u = 2t+3 \\ dv = e^t dt \end{array} \right. \quad \left| \begin{array}{l} du = 2\,dt \\ v = e^t \end{array} \right.$

$$I = \int_{-1}^1 (2\,t+3)\, e^t\, dt = \Big[(2t+3)e^t\Big]_{-1}^1 - 2\int_{-1}^1 e^t\, dt$$
$$= \Big(5\,e\Big) - \Big(e^{-1}\Big) - 2\Big[e^t\Big]_{-1}^1 = 5\,e - e^{-1} - 2\left(e - e^{-1}\right)$$

$$\boxed{I = 3\,e + \frac{1}{e}}$$

7) $$I = \int_2^3 \frac{t+2}{t^2-1}\,dt = \int_2^3 \frac{1}{2}\left(\frac{3}{t-1} - \frac{1}{t+1}\right)\,dt = \left[\frac{3}{2}\ln(t-1) - \frac{1}{2}\ln(t+1)\right]_2^3$$

$$I = \left(\frac{3}{2}\ln(2) - \frac{1}{2}\ln(4)\right) - \left(\frac{3}{2}\ln(1) - \frac{1}{2}\ln(3)\right) \qquad \boxed{I = \frac{1}{2}\ln(6)}$$

8) $$I = \int_0^1 \frac{8\,t+14}{(t+1)(t-2)(t+3)}\,dt = \int_0^1 \left(\frac{-1}{t+1} + \frac{2}{t-2} - \frac{1}{t+3}\right)\,dt$$

$$I = \Big[-\ln(t+1) + 2\ln(2-t) - \ln(t+3)\Big]_0^1 = \Big(-\ln(2) + 0 - \ln(4)\Big) - \Big(0 + 2\ln(2) - \ln(3)\Big)$$

$$\boxed{I = \ln\left(\frac{3}{32}\right)}$$

9) $$I = \int_0^\pi \sin(3t)\,e^{-t}\,dt$$

Par partie : $\left|\begin{array}{l} u = \sin(3t) \\ dv = e^{-t}\,dt \end{array}\right.$ $\left|\begin{array}{l} du = 3\cos(3t)\,dt \\ v = -e^{-t} \end{array}\right.$ 2 fois $\left|\begin{array}{l} u = \cos(3t) \\ dv = e^{-t}\,dt \end{array}\right.$ $\left|\begin{array}{l} du = -3\sin(3t)\,dt \\ v = -e^{-t} \end{array}\right.$

$$\begin{aligned} I = \int_0^\pi \sin(3t)\,e^{-t}\,dt &= \Big[-\sin(3t)e^{-t}\Big]_0^\pi + 3\int_0^\pi \cos(3t)\,e^{-t}\,dt \\ &= \Big(0\Big) - \Big(0\Big) + 3\left(\Big[-\cos(3t)e^{-t}\Big]_0^\pi - 3\int_0^\pi \sin(3t)\,e^{-t}\,dt\right) \\ I &= 3\Big(\big(e^{-\pi}\big) - \big(-1\big) - 3I\Big) \\ I &= 3\big(e^{-\pi}+1\big) - 9I \\ 10\,I &= 3\big(e^{-\pi}+1\big) \end{aligned}$$

$$\boxed{I = \frac{3}{10}\big(e^{-\pi}+1\big)}$$

10) $$I = \int_1^e \frac{1}{t\,\big(2+\ln(t)\big)}\,dt \qquad \text{On pose :} \quad u = 2+\ln(t) \quad \text{donc :} \quad du = \frac{dt}{t}$$

$$\begin{aligned} I &= \int_1^e \frac{1}{t\,\big(2+\ln(t)\big)}\,dt = \int_2^3 \frac{1}{u}\,du \\ &= \Big[\ln(u)\Big]_2^3 = \Big(\ln(3)\Big) - \Big(\ln(2)\Big) \qquad \boxed{I = \ln\left(\frac{3}{2}\right)} \end{aligned}$$

TS 1 IRIS : Devoir N° 4 Sujet B (Solution)

Calculer les intégrales suivantes

1) $$I = \int_0^1 \left(t + e^t + \frac{1}{2}\right) dt = \left[\frac{t^2}{2} + e^t + \frac{t}{2}\right]_0^1 = \left(\frac{1}{2} + e + \frac{1}{2}\right) - (1) \qquad \boxed{I = e}$$

2) $$I = \int_0^{\frac{\pi}{6}} \sin(3t)\, dt = \left[\frac{-\cos(3t)}{3}\right]_0^{\frac{\pi}{6}} = (0) - \left(\frac{1}{3}\right) \qquad \boxed{I = -\frac{1}{3}}$$

3) $$I = \int_0^{\frac{\pi}{6}} \frac{\sin(t)}{\cos(t)}\, dt \qquad \text{On pose :} \quad u = \cos(t) \quad \text{donc :} \quad du = -\sin(t)dt$$

$$I = \int_0^{\frac{\pi}{6}} \frac{\sin(t)}{\cos(t)}\, dt = \int_1^{\frac{\sqrt{3}}{2}} \frac{-1}{u}\, du$$
$$= \Big[-\ln(u)\Big]_1^{\frac{\sqrt{3}}{2}} = \left(-\ln\left(\frac{\sqrt{3}}{2}\right)\right) - \Big(-\ln(1)\Big) \qquad \boxed{I = -\frac{1}{2}\ln\left(\frac{3}{4}\right)}$$

4) $$I = \int_0^1 \frac{7}{t+2}\, dt = \Big[7\ln(t+2)\Big]_0^1 = 7\ln(3) - 7\ln(2) \qquad \boxed{I = 7\ln\left(\frac{3}{2}\right)}$$

5) $$I = \int_0^1 \frac{t+4}{t^2+4}\, dt = \int_0^1 \left(\frac{1}{2}\frac{2t}{t^2+4} + \frac{4}{t^2+4}\right) dt = \left[\frac{1}{2}\ln(t^2+4) + \frac{4}{2}\arctan\left(\frac{t}{2}\right)\right]_0^1$$

$$I = \left(\frac{1}{2}\ln(5) + 2\arctan\left(\frac{1}{2}\right)\right) - \left(\frac{1}{2}\ln(4) + 2\arctan(0)\right)$$

$$\boxed{I = \frac{1}{2}\ln\left(\frac{5}{4}\right) + 2\arctan\left(\frac{1}{2}\right)}$$

6) $$I = \int_{-1}^1 (3\,t+1)\, e^t\, dt$$

Intégration par partie : $\left| \begin{array}{l} u = 3t+1 \\ dv = e^t dt \end{array} \right. \quad \left| \begin{array}{l} du = 3\,dt \\ v = e^t \end{array} \right.$

$$I = \int_{-1}^1 (3\,t+1)\, e^t\, dt = \Big[(3\,t+1)e^t\Big]_{-1}^1 - 3\int_{-1}^1 e^t\, dt$$
$$= \Big(4\,e\Big) - \Big(-2e^{-1}\Big) - 3\Big[e^t\Big]_{-1}^1 = 4\,e + 2e^{-1} - 3\left(e - e^{-1}\right)$$

$$\boxed{I = e + \frac{5}{e}}$$

7) $$I = \int_2^3 \frac{t-3}{t^2-1}\,dt = \int_2^3 \left(\frac{-1}{t-1} + \frac{2}{t+1}\right)\,dt = \Big[-\ln(t-1) + 2\ln(t+1)\Big]_2^3$$

$$I = \Big(-\ln(2) + 2\ln(4)\Big) - \Big(-\ln(1) + 2\ln(3)\Big) \qquad \boxed{I = \ln\left(\frac{8}{9}\right)}$$

8) $$I = \int_0^\pi \cos(2t)\,e^{-t}\,dt$$

Par partie : $\left|\begin{array}{l} u = \cos(2t) \\ dv = e^{-t}\,dt \end{array}\right|$ $\left|\begin{array}{l} du = -2\sin(2t)\,dt \\ v = -e^{-t} \end{array}\right.$ 2 fois $\left|\begin{array}{l} u = \sin(2t) \\ dv = e^{-t}\,dt \end{array}\right|$ $\left|\begin{array}{l} du = 2\cos(2t)\,dt \\ v = -e^{-t} \end{array}\right.$

$$\begin{aligned} I = \int_0^\pi \cos(2t)\,e^{-t}\,dt &= \Big[-\cos(2t)e^{-t}\Big]_0^\pi - 2\int_0^\pi \sin(2t)\,e^{-t}\,dt \\ &= \Big(-e^{-\pi}\Big) - \Big(-1\Big) - 2\left(\Big[-\sin(2t)e^{-t}\Big]_0^\pi + 2\int_0^\pi \cos(2t)\,e^{-t}\,dt\right) \\ I &= 1 - e^{-\pi} - 2\Big((0) - (0) + 2I\Big) \\ &= 1 - e^{-\pi} - 4I \\ 5\,I &= 1 - e^{-\pi} \end{aligned}$$

$$\boxed{I = \frac{1-e^{-\pi}}{5}}$$

9) $$I = \int_{-1}^1 \frac{6\,t-8}{(t+2)(t-2)(t-3)}\,dt = \int_{-1}^1 \left(-\frac{1}{t+2} - \frac{1}{t-2} + \frac{2}{t-3}\right)\,dt$$

$$I = \Big[-\ln(t+2) - \ln(2-t) + 2\ln(3-t)\Big]_{-1}^1 = \Big(-\ln(3) - 0 + 2\ln(2)\Big) - \Big(0 - \ln(3) + 2\ln(4)\Big)$$

$$\boxed{I = -2\ln(2)}$$

10) $$I = \int_1^e \frac{1}{t\,\big(\ln(t)+3\big)}\,dt \qquad \text{On pose :} \quad u = \ln(t) + 3 \quad \text{donc :} \quad du = \frac{dt}{t}$$

$$\begin{aligned} I &= \int_1^e \frac{1}{t\,\big(\ln(t)+3\big)}\,dt = \int_3^4 \frac{1}{u}\,du \\ &= \Big[\ln(u)\Big]_3^4 = \Big(\ln(4)\Big) - \Big(\ln(3)\Big) \qquad \boxed{I = \ln\left(\frac{4}{3}\right)} \end{aligned}$$